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BOILING AND BUBBLING HEAT TRANSFER UNDER FREE CONVECTION OF LIQUID

S. S. KUTATELADZE
Institute of Thermophysics, Siberian Branch of the USSR Academy of Sciences,
Novosibirsk, U.S.S.R.

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Abstract—Experimental data are considered, the fundamental similarity numbers are introduced and the integral laws of heat transfer and detachment effect with free convection in liquid pool-boiling are established.

NOMENCLATURE

c,	specific heat of liquid [J/kg K];
D,	diameter of heater, bubble [m];
g,	acceleration [m/s^2];
j,	mass velocity [kg/m^2 s];
P,	pressure [N/m^2];
v,	kinematic viscosity [m^2/s];
q, q_{cr},	heat flux, critical heat flux [W/m^2];
R,	radius [m];
r,	latent heat [J/kg];
T,	temperature [K];
t,	time [s];
U', U'', U''_{cr},	velocity of liquid flow, bubbling velocity and critical velocity of bubbling [m/s];
α,	heat-transfer coefficient [W/m^2 °C];
ΔT,	difference between heating surface and saturation temperatures;
δ_*,	linear scale of capillary disturbances from formula (1) [m];
ϑ,	subcooling of the bulk of liquid below saturation temperature;
λ,	thermal conductivity of liquid[W/m °K];
ρ', ρ'',	densities of liquid and gas (vapor) [kg/m^3];
σ,	coefficient of surface tension [N/m[;
θ,	contact angle.

INTRODUCTION

THE PROBLEM of boiling is one of the most, if not the most, complicated problem in the heat-transfer theory. A formal mathematical model [1–3] makes it possible to identify the similarity numbers (criteria) that govern this process. However, as of now no closed analytical solution exists for the intensive turbulent heat transfer with generation of vapor bubbles and their departure from the heating surface. Also, the number of pertinent similarity criteria is so great that, if all possible effects are to be taken into account, the clarity and validity of the empirical correlations of the experimental data become obscure. In such situations, of special importance become semi-empirical models that permit isolation of the main parameters of the process and construction of relatively simple relationships.

For the nucleate boiling heat transfer and liquid detachment from the heating surface (occurrence of film boiling), of primary importance are the hydrodynamic processes [1–3]. The behaviour of the gas component is also of consequence [4, 5]. With a certain approximation, the hydrogasdynamic aspects of boiling heat transfer may be simulated by bubbling gas through a microporous surface [2, 5].

The main object in the study of the physics of boiling heat transfer is a pool of freely convecting saturated liquid.

Systematic studies of the mechanism of liquid boiling in connection with the problems of thermohydrodynamics were initiated in the 1930's by Jacob in Germany, Nikiyama in Japan, the group headed by the author in the Soviet Union, McAdams, Craider *et al.* in the United States.

Nikiyama [6] was the first to obtain a "boiling curve" for a pool of freely convecting saturated water at atmospheric pressure and to clearly show the existence of the two basic boiling regimes, namely the nucleate and film boiling.

Jacob [7] suggested that for a characteristic linear dimension of the boiling process one may use the capillary constant

$$\delta_* = \left[\frac{\sigma}{g(\rho' - \rho'')}\right]^{1/2} \tag{1}$$

and an appropriate modification of the Nusselt number

$$Nu_* = \frac{\alpha}{\lambda}\delta_*. \tag{2}$$

Craider and Gilliland, in their experiments [8], vividly demonstrated the effect of the number of nucleation sites on the heating surface and the amount of air absorbed in it on the absolute level of boiling heat transfer.

The author of the present paper has introduced the fundamental number of heat transfer for the matter undergoing a change in the state of aggregation [9]

$$K = \frac{r}{c\Delta T}. \tag{3}$$

In the experiments carried out by the author in collaboration with L. M. Zysina-Molozhen it has been shown that the vapour bubble growth rate scale, introduced by Jacob and equal to the departure diameter by the nucleation rate product, is an internal function of the boiling process decreasing almost inversely to pressure [10]. These experiments revealed for the first time that the vapour bubble departure diameter was not exactly proportional to the capillary constant. Based on experiments with a large variety of liquids, Cichelli and Bonilla [11] show that the curve of the critical heat flux, at which the film boiling occurs, vs pressure has a maximum at about 1/3 the thermodynamic critical pressure of the given medium.

Rosenow [12] has paid attention to the fact that impetus for boiling is the mass rate of vaporization

$$j'' = \frac{q}{r} \tag{4}$$

and accordingly modified the Reynolds number [11] to

$$Re_* = \frac{j''\delta_*}{\mu}. \tag{5}$$

A respective form for the Peclet number is

$$Pe_* = \frac{cj''\delta_*}{\lambda}. \tag{6}$$

Below, a brief discussion is given of the basic results obtained in the investigations of the integral effects of boiling heat transfer which this author has been carrying out for many years. In different periods of time, the following research workers have been taking part in these studies: B. P. Avksentyuk, G. I. Bobrovich, V. M. Borishansky, I. I. Gogonin, L. M. Zysina-Molozhen, A. I. Leontiev, I. G. Malenkov, N. N. Mamontova, V. N. Moskvicheva, L. L. Shneiderman, L. S. Shtokolov and others.

The problems related to the effect of surface roughness, average wetting angle, thermodynamic properties of the wall etc., dealt both in our and in other numerous publications, have been left beyond the scope of the present paper for these reasons.

HEAT TRANSFER FROM A MICROPOROUS SURFACE IN BUBBLING GAS THROUGH LIQUID

The experiments show that with the size of the pores below 0.2 mm and population density above 400 per cm^2, the relationships that describe the bubbling heat-transfer intensity and the effect of liquid detachment from the bubbling surface remain virtually unchanged. All of the results given in this section were obtained under these conditions referred to as bubbling through a microporous surface. In studying the effect of the rate of nitrogen bubbling through water on the heat-transfer coefficient, three main regions have been found experimentally: (1) weakly-developed bubbling when there is a slow increase in heat transfer intensity with growth of the bubbling rate, (2) developed bubbling when, on the average, $\alpha \sim U''^{2/3}$, (3) quasi-stationary heat transfer [5]. Moreover, the region of the developed bubbling expands with increasing water temperature, i.e. with increase in liquid evaporation into gas bubbles. Therefore, all subsequent experiments were conducted at about 5°C, i.e. when the developed nucleate bubbling region was least extended.

In the experiments on determining the function $\alpha(q)$ for water boiling on graphite rods, one clearly distinguishes the regions of intensive increase in the heat-transfer coefficient and its practical stabilization [13]. However, the latter region is usually not largely extended, being markedly metastable due to evaporation-induced destruction of liquid films.

A quantitative analogy is most clearly observed in the case of developed nucleate bubbling and boiling when, on the average, one may assume that for

$$\begin{aligned} &\text{bubbling } \alpha \sim U''^{2/3} \\ &\text{boiling } \alpha \sim q^{2/3}. \end{aligned} \tag{7}$$

This analogy will show up more vividly if the heat flux density is replaced by the rate of evaporation ("bubbling" in the process of boiling)

$$U'' = \frac{q}{r\rho''}. \tag{8}$$

The fundamental results obtained in the experiments on bubbling are those which establish the existence of different effects of a change in pressure and molecular weight and of an extensive region of self-similarity with respect to the viscosity of liquid. Just as in boiling, a change in density due to a pool pressure increase resulted in the growth of the heat-transfer coefficient at the same linear velocity of bubbling. Variations in the gas molecular weight had no effect within the experimental accuracy. All experimental results, in this case, were well generalized by

$$\alpha = f(P\,U''), \tag{9}$$

which is directly evident from Fig. 1.

This result correlates well with the similarity number suggested by the author many years ago [10] and which presently may be considered as the ratio of the boiling Peclet number to the squared modification of the Mach number based on the ratio of the propagation velocity scale of capillary disturbances and the isothermal speed of sound in the gas component

$$M_*^2 = \frac{\rho''}{P} g\,\delta_* \tag{10}$$

$$K_* = \frac{Pe_*}{M_*^2} = \frac{cU''P}{g\lambda}. \tag{11}$$

Figure 2 shows the relationship

$$Nu_* = f\left(\frac{Pe_*}{M_*^2}\,;\ \frac{U''\mu'}{\sigma}\right) \tag{12}$$

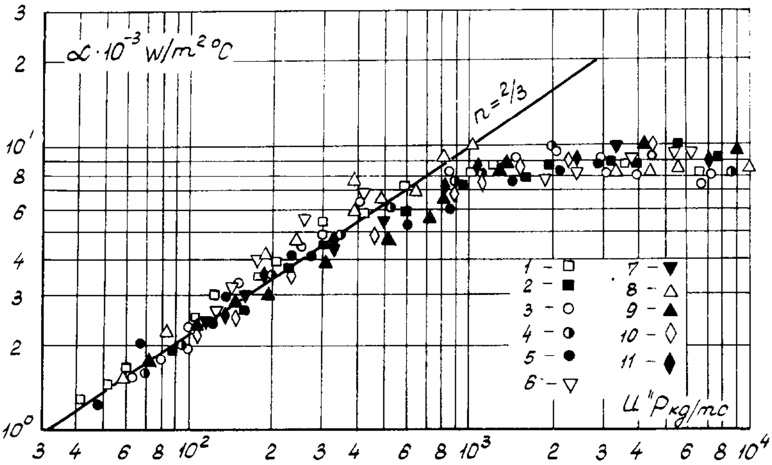

FIG. 1. Relation $\alpha = f(PU'')$ for bubbling of different gases into water through porous plates with $\varphi_1 = 0.2$ and 0.44 at pressures [5]: Argon: (1) $9.8\times10^4\,\mathrm{N/m^2}$; (2) $68.7\times10^4\,\mathrm{N/m^2}$; Nitrogen: (3) $9.81\times10^4\,\mathrm{N/m^2}$; (4) $68.7\times10^4\,\mathrm{N/m^2}$; (5) $166.8\times10^4\,\mathrm{N/m^2}$; Helium: (6) $9.81\times10^4\,\mathrm{N/m^2}$; (7) $39.2\times10^4\,\mathrm{N/m^2}$; Hydrogen: (8) $9.81\times10^4\,\mathrm{N/m^2}$; (9) $68.7\times10^4\,\mathrm{N/m^2}$. Through plate with $\varphi_1 = 0.13$, Argon: (10) $9.81\times10^4\,\mathrm{N/m^2}$; Hydrogen: (11) $9.81\times10^4\,\mathrm{N/m^2}$.

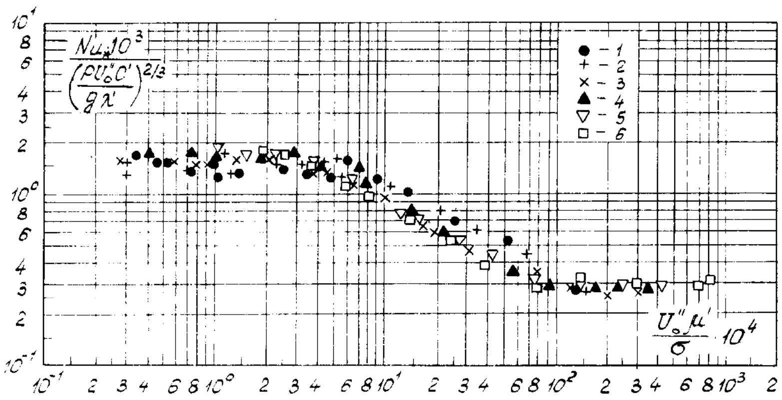

FIG. 2. Relation $Nu_*(\lambda' g/cPU''_0)^{2/3} = f(U''_0\mu'/\sigma)$. (1) distilled water. Water-glycerine solutions [5]: (2) 5%; (3) 10%; (4) 20%; (5) 30%; (6) 50%.

obtained from the experiments on bubbling of hydrogen, helium, nitrogen and argon at pressures from 1 to 40 abs atm into water and water–glycerine solutions through a microporous surface. Two self-similar and transient regions are clearly visible here. The experimental data for the first region are correlated by

$$Nu_* = 1.5\times10^{-3}K_*^{2/3}. \tag{13}$$

This region is analogous to the developed nucleate boiling. The relevant experimental data will be considered in what follows.

The relation between q and α shows that at the preassigned temperature difference, ΔT, the similarity criterion

$$\frac{c\Delta TP}{gr\rho''\delta_*} = K^{-1}M_*^{-2} \tag{14}$$

is identical to similarity number (11) and the relation

$$Nu_* \sim K^{-2}M_*^{-4} \tag{15}$$

is identical to formula (13).

Numerous data show that, depending on the state of the heater surface and the experimental conditions, the exponents of q and ΔT change, for example, as [37, 38]:

$$\left.\begin{array}{ll}\alpha\sim q^n; & 0.6<n<0.8;\\ \alpha\sim\Delta T^m; & 3/2<m<4.\end{array}\right\} \tag{16}$$

DETACHMENT OF A LIQUID FROM A MICROPOROUS SURFACE BY BUBBLING OF GAS

The case of pure (i.e. without heat transfer) hydrodynamic detachment of liquid by a gas has been studied. The detachment effect was fixed by disappearance of the electricity conduction between a metallic microporous plate and a thin wire located parallel to the plate at 0.1 mm above it. The working liquids were water, ethanol and water-glycerine mixtures. The working gases were hydrogen, helium, nitrogen, argon and xenone. The pressure ranged within 1–40 abs atm.

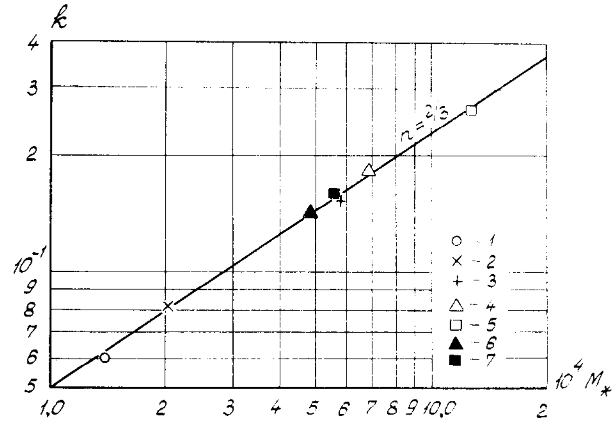

FIG. 3. Variation of K vs M_*. Bubbling of (1) hydrogen; (2) helium; (3) nitrogen; (4) argon; (5) xenone through water. Bubbling of (6) nitrogen; (7) argon [14] through ethanol.

The fundamental similarity criterion for the detachment effect under the conditions of free convection is obtained from the relation of the scales of the kinetic energy of the gas, which separates liquid from the surface in bubbling, and of the gravitational force acting on the suspended liquid

$$U''^2\rho'' \sim g(\rho'-\rho'')\delta_*. \tag{17}$$

This yields the similarity criterion for the detachment of liquid in bubbling

$$k = \frac{U''_{cr}(\rho'')^{1/2}}{[g\sigma(\rho'-\rho'')]^{1/4}} \tag{18}$$

and for the generation of the vapor film in boiling

$$k = \frac{q_{cr}}{r(\rho'')^{1/2}[g\sigma(\rho'-\rho'')]^{1/4}}. \tag{19}$$

For incompressible nonviscous components and rather extended surfaces the following condition is valid

$$k = \text{const.} \tag{20}$$

In the experiments with water and ethanol, the following, ensuing from (20), condition was valid for each type of the gas at the given temperature

$$U''_{cr}(\rho'')^{1/2} = \text{const.} \tag{21}$$

However, the constants for each gas differed substantially. Moreover, a monotonous increase in the parameter (21) with the rise in the molecular gas weight was observed. As a working hypothesis, this effect may be attributed to the M_* number which at the given temperature is directly proportional to the molecular weight of the gas.

The influence of viscosity may be characterized by the modified Archimedes number

$$Ar_* = \frac{g\delta_*^3}{\nu^2}\left(1-\frac{\rho''}{\rho'}\right). \tag{22}$$

The experiments with water–glycerine mixtures show that the detachment effect is self-similar with respect to viscosity at $Ar_* > 1.10^4$.

Figure 3 gives the function $k(M_*)$ for the self-similar region which may be described by

$$k = 30M_*^{2/3}. \tag{23}$$

The behaviour of the function $k(M_*; Ar_*)$ is shown in Fig. 4 [14].

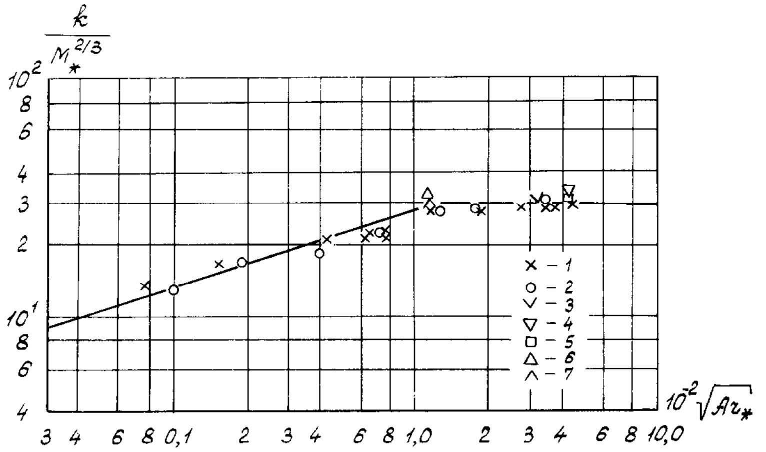

FIG. 4. Variation of viscosity vs k [14]. Water–glycerine mixtures of different concentration (from 5 to 80%). (1) Nitrogen; (2) helium. Distilled water: (3) xenone; (4) argon; (5) nitrogen. Ethanol: (6) argon; (7) nitrogen.

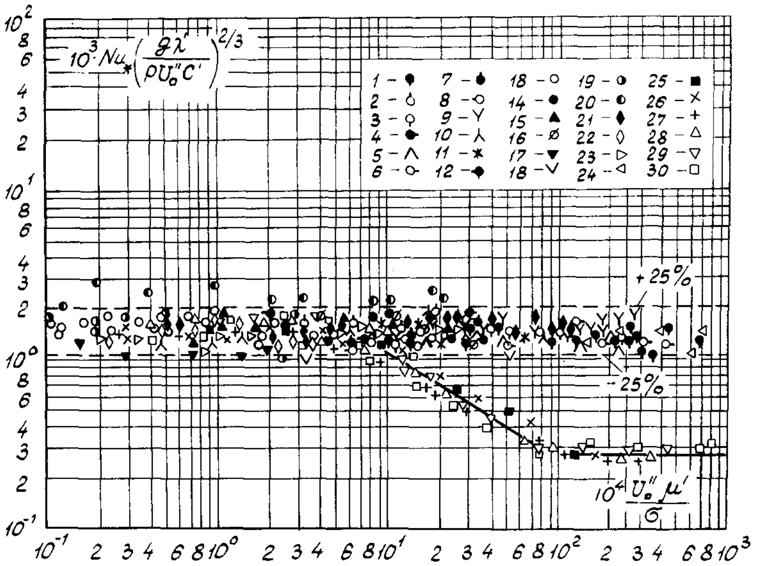

FIG. 5. Variation of heat-transfer coefficient in the coordinates of equation (25) [5]. Boiling of sodium: (1)–(5), $(4 \div 14) \times 10^4\,N/m^2$; potassium: (6)–(8) $(6.57 \div 11.75) \times 10^4\,N/m^2$; cesium: (9) $(2.45 \div 15.7) \times 10^4\,N/m^2$; water: (13)–(16) $(9.81 \div 98.1) \times 10^4\,N/m^2$; benzene: (17), (18) $(9.81 \div 20.7) \times 10^4\,N/m^2$; nitrogen (on stainless steel): (19) $(9.81 \div 98.1) \times 10^4\,N/m^2$; nitrogen (on copper): (20) $(9.81 \div 98.1) \times 10^4\,N/m^2$; helium: (21) $(1.23 \div 5.29) \times 10^4\,N/m^2$; Freon-12: (22) $(20.6 \div 92.8) \times 10^4\,N/m^2$; Freon-21: (23), (34) $(14.7 \div 98.1) \times 10^4\,N/m^2$. Bubbling, $P = 9.81 \times 10^4\,N/m^2$; (25) distilled water; aqueous solutions of glycerine: (26) 5%; (27) 10%; (28) 20%; (29) 30%; (30) 50%.

Thus, the experiments on heat transfer and liquid detachment from the microporous surface by bubbling the gas have revealed the effect of compressibility of the gas component on the thermohydrodynamics of the gas–liquid mixtures even at low velocities of the main flow. This, naturally, has entailed new unsolved problems. Thus, relation (23) does not show the possibility for liquid–liquid bubbling though the difference between the liquid–gas and liquid–liquid systems is observable in bubbling columns [15].

SATURATED LIQUID BOILING HEAT TRANSFER

Formula (13), re-written for the process of boiling by incorporating (8), has the following form in primary quantities (at $\rho'' \ll \rho'$)

$$\frac{\alpha}{\lambda}\left(\frac{\sigma}{g\rho'}\right)^{1/2} \sim \left(\frac{cqP}{gr\lambda\rho''}\right)^{2/3}. \qquad (24)$$

Figure 5 gives numerous data for several liquids in the coordinates

$$\left\{Nu_* K_*^{-2/3};\ \frac{U''\mu'}{\sigma}\right\}. \qquad (25)$$

As is seen, the self-similarity with respect to viscosity is observed for all the liquids in boiling, while in bubbling its disturbance occurs at $(U''\mu'/\sigma) > 10^{-4}$. This seems to be associated with liquid evaporation into vapor bubbles and in some manner be affected (beyond the self-similarity region) by K_* or the Jacob number

$$Ja = \frac{\rho'}{\rho''}K^{-1}. \qquad (26)$$

Thus it may be assumed that the correlation

$$Nu_* \sim K_*^n \qquad (27)$$

is fundamental for the purpose of describing the developed nucleate boiling heat transfer.

FIRST HYDRODYNAMIC CRISIS IN SATURATED LIQUID

Figure 6 presents several experimental data on the first critical heat flux q_{cr_1} at which a transition from metastable to developed film boiling occurs. At $L > \delta_*$ this process is virtually self-similar with respect to the characteristic dimension of the heating surface and is purely hydrodynamic at about atmospheric and higher pressures. A more accurate self-similarity criterion relative to the thermal and hydrodynamic conditions which occur at low pressures will be discussed in the next section. The same figure illustrates formula (23) derived from data on bubbling. As is seen, in general a qualitative agreement is observed, but the experimental points are much spread and for most of them average value of k is close to that reported in literature, i.e. 0.13–0.16. It is likely that intensive evaporation diffuses the effect of M_* on liquid detachment. In any event, this relation requires refinement since formula (19) gives $q_{cr} \sim g^{1/4}$, while formula (23) yields $q_{cr} \sim g^{5/12}$. Most of the experimental data for the effect of acceleration on initiation of film boiling yet correspond, on the average, to the first of these relations. Here, we may mention the results obtained by D. A. Labuntsov and R. I. Soziev for five orders of acceleration [16]. However, according to Kirichenko *et al.* [17] the exponent of g varies within 0.2–0.4 and probably

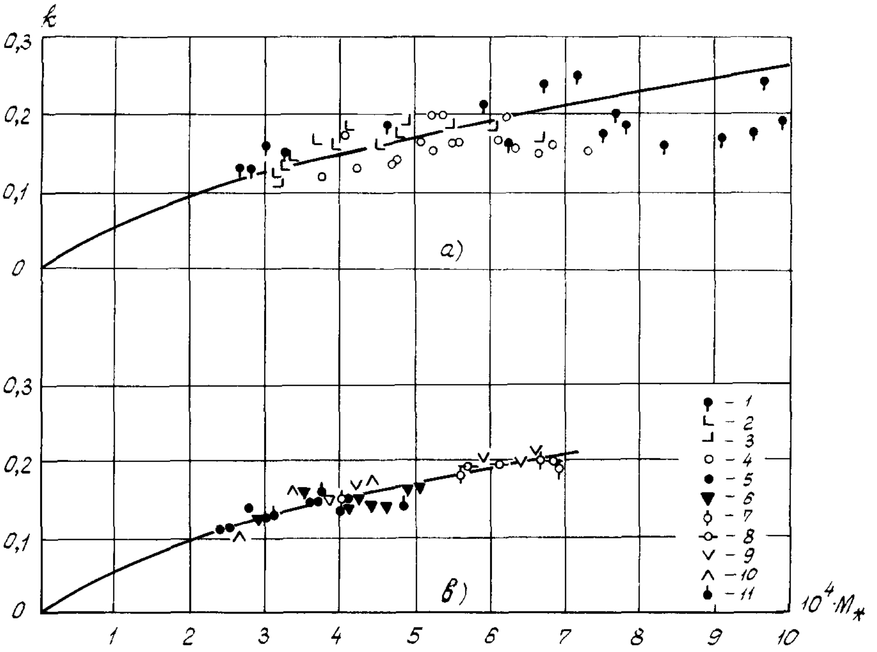

FIG. 6. Variation of k vs M_*. Boiling of cryogenic (1)–(4) and standard (5)–(11) liquids at different pressures [14]. (1) helium; (2) hydrogen; (3) argon; (4) nitrogen; (5) water; (6) ethanol; (7) benzene; (8) heptane; (9) pentane; (10) propane; (11) methanol.

depends on the non-equilibrium temperature field that may appear in such experiments.

The effect of subcooling of the bulk of the liquid below the saturation temperature is well incorporated in the linear approximation [2]

$$q_{cr} = q_{cr,0} + q', \tag{28}$$

where $q_{cr,0}$ is the critical heat flux from the boundary of the vapor film into the cold liquid core.

If for the transfer of heat from the interface, which is continuously replenished by evaporation, we adopt the model of unsteady heat conduction into an infinite space [18], then

$$q' \sim \vartheta(\lambda c\rho'/t)^{1/2}, \tag{29}$$

where $\vartheta = T'' - T_0$ is the difference between the temperature of saturation and that in the bulk of the liquid outside the thermal boundary layer.

Since free convection continuously generates capillary waves on the surface of the vapor film, the characteristic time scale is equal to

$$t \sim \frac{\delta_*}{U''} \sim \frac{r\rho''\delta_*}{q_{cr,0}}, \tag{30}$$

from which it follows that

$$\bar{q}_{cr} = 1 + \zeta K^{-1} Pr^{-1/2} Ar_*^{-1/4} \left(\frac{\rho'}{\rho''}\right)^{3/4}, \tag{31}$$

where ζ is a certain constant.

For many liquids, the complex $Pr^{1/2} Ar_*^{1/4}$ is almost constant. Thus, at $0.01 < (P/P_{cr}) < 0.5$ this complex for water, benzene, ethanol is respectively equal to 34–35, 45–55, 35–47. In these cases one may use the simplified formula

$$\bar{q}_{cr} = 1 + 0.1 \left(\frac{\rho'}{\rho''}\right)^{3/4} K^{-1}. \tag{32}$$

Here $\bar{q}_{cr} = q_{cr}/q_{cr,0}$ is a relative change in the critical heat flux.

Zuber *et al.* [19] considered one of the specific models of the structure of an incompressible two-phase wall layer (a spherical vapor bubble in a liquid cube) and found that $k_0 \approx \pi/4$ and $\zeta = 5.33$. The experimental data given in [21, 22, 32, 33] are fairly well fitted by relations (31) and (32).

Experimental data on the effect of the diameter of an extended cylindrical heater on the critical heat flux in a freely convecting saturated liquid show that at $D < \delta_*$, not only the dimensions of the heater are of significance but also its orientation with respect to the gravitational force [20].

A generalized experimental chart of the expression

$$\bar{q}\left[K^{-1} Pr^{-1/2} Ar_*^{-1/4} \left(\frac{\rho'}{\rho''}\right)^{3/4}; \frac{D}{\delta_*}\right] \tag{33}$$

is given in Fig. 7.

A decrease in the critical heat flux at $D < 0.5\delta_*$ is caused by blanketing of separate sections of the heater with bubbles and associated drastic deterioration of the local heat transfer. The resultant superheat causes damage of the heating surface which is identified with the onset of the heat transfer crisis [21, 22].

BOILING AT EXTREMELY LOW PRESSURES

In the process of boiling at very low pressures, two main phenomena can be distinguished, i.e. depletion

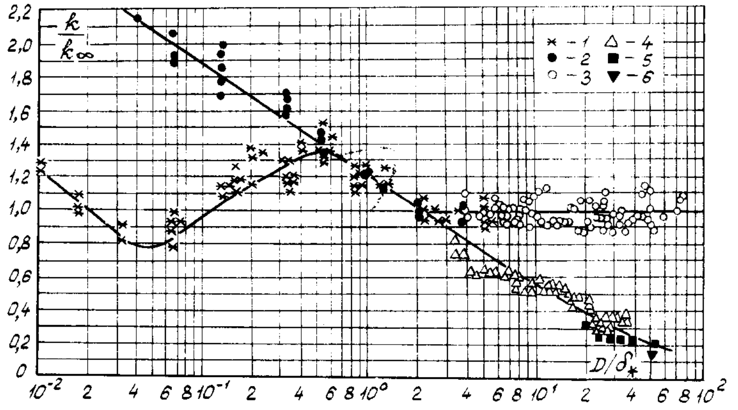

FIG. 7. Generalization of experimental data in the coordinates of equation (33).
(1) cylindrical heaters, ethanol $t = t''$
(2) cylindrical heaters, ethanol $K^{-1}(\rho'/\rho'')^{3/4} \geq 18$ } [21];
(3) plate thermally insulated from below
(4) plate thermally insulated from above } ethanol [22];
(5) section with thermal insulation from below, helium [34];
(6) section with thermal insulation from above, helium [35].

of the potential nucleation sites from the heating surface and increase in the vapor bubble departure diameter. The order of the critical radius of the vapor bubble nucleus is described by the known formula

$$R_{cr} = \frac{2\sigma T''}{r\rho''(T_w - T'')}, \tag{34}$$

where T_w is the wall temperature, T'' is the saturation temperature above the surface. Microcavities on the heating surface, the size distribution of which depends on the finish of the surface, serve as nucleation sites in boiling. At $P \to 0$, $\rho'' \to 0$ and $R_{\min} \to \infty$, i.e. at a certain pressure, the geometrical nucleation sites may be totally idle. As a result, an appreciable superheating of liquid may occur with its subsequent transition into a metastable state. In this state, the liquid may spontaneously boil-up with the effect that, this process being developed, a single-phase convection regime may directly transform into the film boiling (bumping) regime by-passing the phase of developed nucleate boiling [23–26]. The mechanism of this transition may be characterized as cavitational, since generation of microbubbles in the bulk of the superheated liquid occurs in the bump-like manner, the wall layer evaporates and the vapor film forms. The critical heat fluxes, characteristic of this unstable boiling, lie in the range between the first critical value, $q_{cr,1}$, when there is a conventional transition from the developed nucleate to film boiling, and the second critical value, $q_{cr,2}$, when a reverse transition takes place. We shall call this phenomenon the third boiling crisis for which the following inequality holds

$$q_{cr,2} < q_{cr,3} < q_{cr,1}. \tag{35}$$

Let us assume that the cavitational boiling in the vicinity of the heater is induced by the momentum introduced by the bubble originated spontaneously in the wall superheated liquid layer. The appropriate criterion may be written as [26]:

$$\frac{\sigma R_{cr}^2}{\rho' v_*^2 V_*'}. \tag{36}$$

Here v_* is the characteristic velocity of the interface displacement and V_*' is the virtual liquid volume entrained by the interface. Boiling-up of liquid volume may be described by formula (29) and by

$$v_* = \frac{dR}{dt} \sim \frac{\lambda c \rho'}{R}\left(\frac{\Delta T}{r\rho''}\right)^2. \tag{37}$$

From (34), (36) and (37) it follows that the critical temperature difference for the third boiling crisis is described by

$$\Delta T_{cr,3} \sim \frac{\sigma^{5/8} T''^{1/2}}{(c\lambda)^{1/4} \rho'^{3/8} V_*'^{1/8}}. \tag{38}$$

The experiments show that $\Delta T_{cr,3}$ is practically

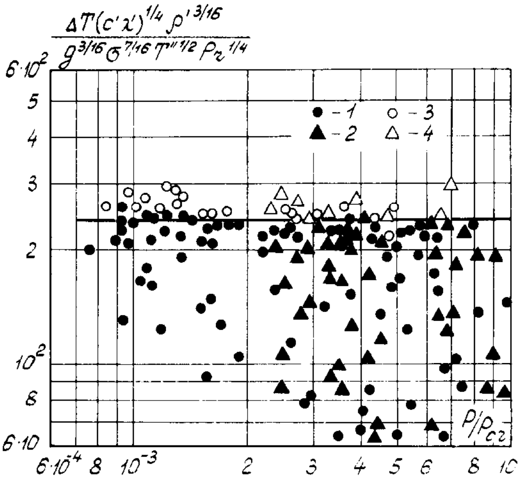

FIG. 8. Processing of experimental data on the superheat preceding boiling-up [26]. (1), (2) transition to nucleate boiling for ethanol and benzene; (3), (4) transition to film boiling for ethanol and benzene.

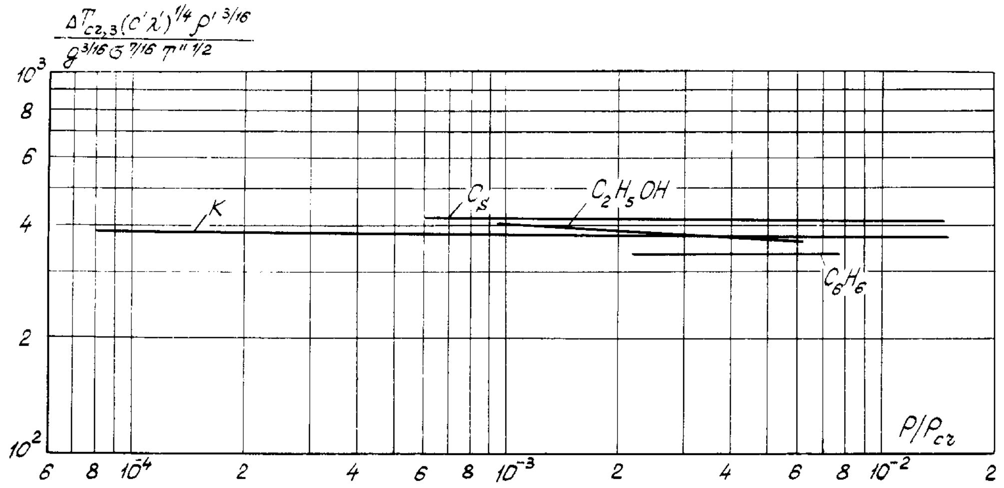

FIG. 9. Minimum superheats for boiling-up to result in film boiling (processing of experimental results) [26].

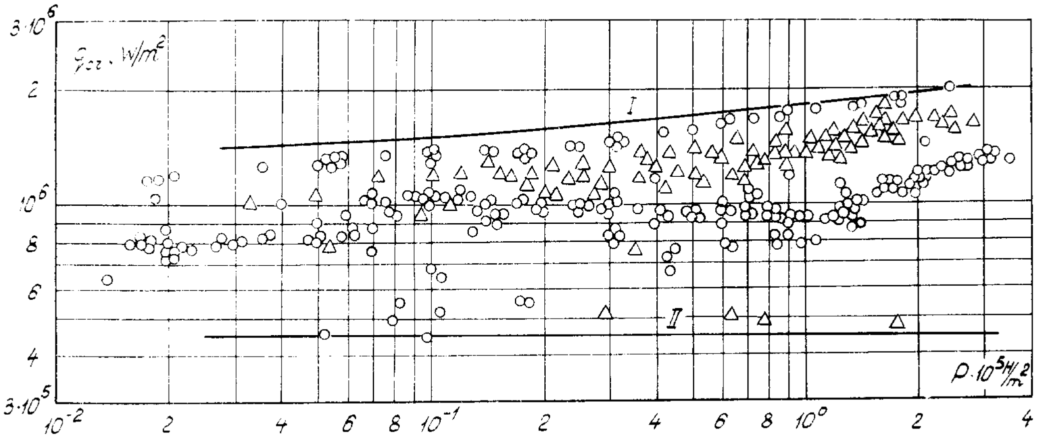

FIG. 10. Experimental data on q_{cr} in cesium boiling. (1) data of [36]; (2) data of [26]; I, calculation by (42); II, calculated curve at $\Delta T_{cr,3} = 3.8 \times 10^2$ for minimal heat fluxes at which boiling-up leads to film boiling.

constant or changes slightly with pressure. It means that the characteristic volume V_*' is determined by the linear parameter δ_* rather than R_{cr}. Then the criterion of film boiling initiated from the metastable wall layer of liquid takes the form

$$\Delta\tilde{T}_{cr,3} = \frac{(c\lambda)^{1/4}\rho'^{3/16}\Delta T_{cr,3}}{g^{3/16}\sigma^{7/16}T''^{1/2}}. \tag{39}$$

Figure 8 presents the data on boiling-up temperatures for ethanol and benzene. The demarcation line separates the region of the normal nucleate boiling. The same demarcation lines, but for media whose Prandtl numbers differ by several orders, are shown in Fig. 9.

Under the conditions studied, the quantity q' in (28) is not zero even if the liquid is heated to the saturation temperature at a large distance from the wall.

Assuming, approximately, that

$$q' \sim \Delta T_{cr,3}(\lambda c\rho'/t)^{1/2} \tag{40}$$

and that the characteristic time is determined by purely capillary disturbances

$$t \sim \frac{\delta_*^{1/2}}{g^{1/2}} \tag{41}$$

we obtain the formula ($\rho' \gg \rho''$)

$$q_{cr,1} \approx 0.14\, r\, \rho''^{1/2}(g\sigma\rho')^{1/4} + Cg^{9/16}T''^{1/2}(\lambda c)^{1/4}\sigma^{5/16}\rho'^{7/16}. \tag{42}$$

From the available limited data for liquid metals it is found that $C \approx 170$ and for non-metallic liquids that $C \approx 50\,Pr^{-1/4}$.

Figure 10 gives the relevant data on boiling of cesium.

GROWTH RATE AND DEPARTURE DIAMETER OF A VAPOR BUBBLE

When a vapor bubble is formed on a solid wall, the heat is transferred to it mainly through a thin wall layer (Fig. 11). This process may be regarded as quasi-stationary in contrast to unsteady heat transfer in the bulk of the liquid described by formula (29). The thickness of the liquid layer beneath the bubble

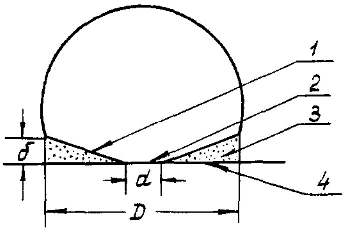

FIG. 11. Scheme of vapor bubble-heating surface contact. (1) contact through liquid layer; (2) dry patch; (3) liquid layer; (4) heating surface.

is proportional to its radius and depends on the wall wettability. Assuming that

$$\left\{\frac{\mathrm{d}R}{\mathrm{d}t}=\frac{q'}{r\rho''};\ q'\sim\frac{\lambda\Delta T}{R}\right\} \tag{43}$$

we get

$$\frac{\mathrm{d}R}{\mathrm{d}t}\sim\frac{\lambda\Delta T}{R\rho''r}. \tag{44}$$

From this, at $R \gg R_{cr}$, we obtain the Labuntsov's formula [27]

$$R=\beta_*\left(\frac{\lambda\Delta Tt}{r\rho''}\right)^{1/2} \tag{45}$$

where β_* is the proportionality factor which depends on the surface wettability.

The conditions of the balance of forces at the moment of bubble separation from the wall ($R = R_0$) may be written in the form

$$\frac{4}{3}\pi R_0^3 g(\rho'-\rho'')=\varphi(\theta)2\pi R_0\sigma+\frac{\xi}{2}\pi R_0^2\rho'\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 + \frac{4}{3}\pi R_0^3\left(\rho''+\frac{1}{2}\rho'\right)\frac{\mathrm{d}^2R}{\mathrm{d}t^2}. \tag{46}$$

From (44) it follows that

$$\frac{\mathrm{d}^2R}{\mathrm{d}t^2}=-\frac{1}{R}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2,\ \text{i.e.} \tag{47}$$

at $\rho' \gg \rho''$, both the hydrodynamic resistance to the bubble growth on the wall and the inertial force are proportional to the squared velocity of the bubble growth. This yields two similarity criteria

$$\tilde{D}=D_0\left(\frac{g\rho'}{\sigma}\right)^{1/2} \tag{48}$$

$$\frac{R_0\rho'}{\sigma}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2_{R_0}. \tag{49}$$

Having eliminated R_0 from (49) via (45) gives the following similarity criterion

$$K_{**}=\left(\frac{Ja}{Pr}\right)^2 Ar_*^{-1}. \tag{50}$$

Figure 12 shows the experimental data of [28–31] processed in these coordinates. The region of self-similarity of the number $\tilde{D}$ and the region of its linear relation to the number K_{**} are clearly visible.

The time of bubble formation is

$$t_0=\frac{r\rho''R_0^2}{\beta_*^2\lambda'\Delta T} \tag{51}$$

and the average bubble growth rate is

$$\frac{D_0}{t_0}=\frac{2\beta_*^2\lambda'\Delta T}{r\rho''D_0}. \tag{52}$$

The balance equation for the moment of bubble separation takes the form

$$\tilde{D}_0=A\cdot(1+BK_{**})^{1/2}. \tag{53}$$

Based on the above experimental data the constants $A = 0.25$ and $B = 10^5$ yield

$$\tilde{D}_0=0.25(1+10^5K_{**})^{1/2}. \tag{54}$$

FUNDAMENTAL SIMILARITY CRITERIA

$Ar_* = (g\delta_*^3/\nu^2)[1-(\rho''/\rho')]$ is the Archimedes number based on the linear scale of capillary disturbances. It characterizes the interaction of the forces of gravity, molecular viscosity and surface tension in free gas–liquid structures.

$\tilde{D} = D/\delta_*$ is the relative linear size of a component in the gas–liquid structure (e.g. the heater diameter, bubble separation diameter).

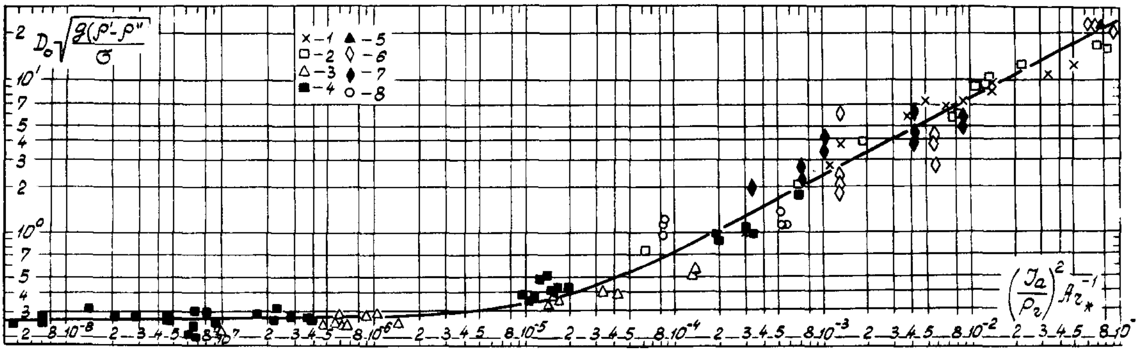

FIG. 12. Processing of experimental data according to vapor bubble diameter. (1) water; (2) ethanol[28]; (3) benzene; (4) ethanol [30]; (5) ethanol [29]; (6) water; (7) methanol; (8) pentane [31].

$Fr_* = U'^2[(\rho'-\rho'')/g\sigma]^{1/2}$ is the modified Froude number. It characterizes the influence of the ordered liquid-phase flow on the gas–liquid structure.

$K = r/\Delta i$ is the thermal similarity criterion at the first-kind phase transitions. It characterizes a relative influence of the latent heat of transition and the enthalpy of superheating (or supercooling) of the given phase with respect to the saturation line (introduced by the author, 1936).

$K_* = cqP/g\lambda r\rho''$ is the criterion of thermal similarity in developed nucleate boiling. It characterizes the heat flux interaction due to the phase transition with the liquid phase thermal conductivity and the influence of the gas phase elasticity on the intensity of transfer processes in the gas–liquid structure (introduced by the author and Zysina-Molozhen [10]). With ΔT preset, the number $K^{-1}M_*^{-2}$ corresponds to this similarity criterion.

$k = q_{cr}/r(\rho'')^{1/2}[g\sigma(\rho'-\rho'')]^{1/4}$ is the criterion of the gas–liquid structure stability (introduced by the author [1]).

$M_* = (\rho''/P)^{1/2}[g\sigma/(\rho'-\rho'')]^{1/4}$ is the modified Mach number. It characterizes the interaction of capillary disturbances at the interface with elastic disturbances in the gas phase (introduced by the author and Malenkov [4]).

$Nu_* = \alpha\delta_*/\lambda$ is the Nusselt number based on the linear scale of capillary disturbances (introduced by Jacob [7]).

$Pe_* = cq\delta_*/\lambda r$ is the modified Peclet number based both on the momentum introduced by the vapor flow and on the linear scale of capillary disturbances.

$Pr = c\mu/\lambda$ is the Prandtl number.

$\Delta\tilde{T}_3 = [(c\lambda)^{1/4}\rho'^{3/16}\Delta T_{cr,3}/g^{3/16}\sigma^{7/16}T''^{1/2}]$ is the similarity criterion characterizing the boundary of the developed (stable) nucleate boiling and unstable boiling (introduced by the author and Avksentyuk, [26]).

ρ''/ρ' is the relative density of phases.

In bubbling without phase transitions, the mass velocity of bubbling $j'' = \rho''U''$ is introduced into the numbers K_*, k and Pe_* instead of the mass velocity of boiling $j'' = q/r\rho''$.

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TRANSFERT THERMIQUE PAR EBULLITION ET PAR BULLAGE AVEC CONVECTION NATURELLE DU LIQUIDE

Résumé—On considère des résultats expérimentaux, on introduit des nombres fondamentaux de similitude et on établit les lois intégrales du transfert de chaleur et de l'effet de détachment avec convection naturelle dans une ébullition de liquide en réservoir.

WÄRMEÜBERGANG BEIM SIEDEN MIT BLASENBILDUNG UNTER DEN BEDINGUNGEN FREIER KONVEKTION DER FLÜSSIGKEIT

Zusammenfassung—Es werden experimentelle Daten betrachtet, die fundamentalen Kennzahlen der Ähnlichkeitstheorie eingeführt und die integralen Gesetze des Wärmeübergangs und des Ablösungseffekts beim Behältersieden mit freier Konvektion aufgestellt.

ТЕПЛООБМЕН ПРИ БАРБОТАЖЕ И КИПЕНИИ В УСЛОВИЯХ СВОБОДНОЙ КОНВЕКЦИИ ЖИДКОСТИ

Аннотация — Рассматриваются экспериментальные данные, вводятся фундаментальные числа подобия и устанавливаются интегральные закономерности теплоотдачи и эффекта оттеснения при свободной конвекции в большом объеме жидкости.